Dear students,

Here is a relatively easy chemistry problem, to calculate the pH of an aqueous solution of NaOH with an initial concentration of $1.0 \times 10^{-8}$ M in NaOH at 25°C.

Why is the title "Wonder in Science" used here? It is not a secret among students at the undergraduate level, unfortunately all over the world, that a very high percentage of students cannot wait to graduate and leave campus. They say that while studying, they do not feel excited about what they are learning; instead they feel bored and worse, they feel an unbearable stress and pressure while trying to meet the requirements for their degrees.

This page is not about to suggest why this is happening or ways to change the direction of the educational system here and abroad in order for students to have fun (yes fun!) while pursuing their degrees.

Graduate students and a very few undergraduates have the opportunity to behold the beauty and wonder in science very often while they are doing their research in a sciences laboratory. This introduction and the solution to the above chemistry problem are to show you the beauty that you can discover by even just solving a routine chemistry problem as an undergraduate student.

Great sages of the world remind us that the highest goal in life, and thus also during college years, is to "KNOW YOURSELF". By pursuing this goal every day you can eventually find the right major, right career and later the right place of work, compatible with your talents and abilities, with your values and ideals, and with your personal needs.

Experiencing tremendous excitement from learning during your undergraduate years is possible! It is a must in order to continue feeling content while working after college. Doing your work now and after college must feel as if you are doing your life's hobbies. Only then will you be truly successful inside and outside. With the exception of a few short periods of doing something that needs to be done and which does not interest you that much, learning should be fun.

The above chemistry problem can be solved in three different ways and amazingly, the answer is identical, pH = 7.02. The assumptions used to solve the problem by the first two methods are different, and the principle used for the third method is also different.

Here is the wonder! How can we get the same answer every time? It is because there is an underlying fundamental principle above and beyond all other principles, equations and forms used. That underlying principle is hard to define, and we might call it "truth or essence in science".

Truth in science does not depend on the nationality, religion, or philosophy of the researcher or the method used to uncover it. The essence of science is eternal,
unmovable and hidden. The fundamental principles of science are rarely revealed as the
history of man has shown, and when they are revealed, they are revealed only to those
that humbly feel the wonder of the physical universe. Albert Einstein was such a humble
man.

I hope that you can feel even a little wonder at our physical universe by solving this
problem in three different ways. First you can solve the problem by two different
methods using a summary table each time. Each method should give you the same value of
\( \text{pH} = 7.02 \).

For the first method when constructing your summary table for your initial
concentrations, assume no self-ionization of water and complete ionization of NaOH. The
second row for the change of concentrations will come from the suppressed self-ionization
of water. With suppressed self-ionized water, \( [\text{OH}^-]_{\text{H}_2\text{O}} = [\text{H}_3\text{O}^+]_{\text{H}_2\text{O}} = y \), the first method
should give you the quadratic equation: \( y^2 + 10^{-8} y - 10^{-14} = 0 \).

For the second method when constructing your summary table for your initial
concentrations, assume self-ionization of water and complete ionization of NaOH before
the system shifts to the new equilibrium position. The second row for the change of
concentrations will come from the shifting of the equilibrium position towards formation
of water. With neutralization and formation of water, \( [\text{OH}^-]_{\text{H}_2\text{O}} = [\text{H}_3\text{O}^+]_{\text{H}_2\text{O}} = y \), the second
method should give you the quadratic equation: \( y^2 - 2.1 \times 10^{-7} y + 10^{-15} = 0 \).

For the third method, you need to construct an equation based on the fact that even
though an aqueous solution contains anions and cations it is, nevertheless, neutral. Neutral
in this case means that the total positive charge is equal to the total negative charge.
With suppressed self-ionized water, \( [\text{H}_3\text{O}^+]_{\text{H}_2\text{O}} = y \), the third method should give you the
quadratic equation: \( y^2 + 10^{-8} y - 10^{-14} = 0 \). The same as in the first method!

Notice that in each case, we find that the total concentration of hydroxide ions is
\( 10.5 \times 10^{-8} \text{ M} \), and because the total concentration of hydronium ions is \( 9.5 \times 10^{-8} \text{ M} \), in each
case the ion product of water is indeed equal to \( 1.0 \times 10^{-14} \).

Finally, notice that in solving this problem, we had to consider the autoionization of water
because in this case, the NaOH solution is of such low concentration \( (1.0 \times 10^{-8} \text{ M}) \) that
water actually contributes to the total concentration of hydroxide ions ten times more
than what NaOH contributes. We could not ignore, therefore, the contribution from
water. Ignoring the contribution from water, a \( 1.0 \times 10^{-8} \text{ M} \) NaOH solution would be
expected to give \( \text{pOH} = 8 \) and thus \( \text{pH} = 6 \). But how would a basic aqueous solution at \( 25^\circ \text{C} \)
have \( \text{pH} < 7 \)? This erroneous result is obtained if we ignore the autoionization of water.

Andreas Toupadakis
Davis, February 15, 2011
Problem

Calculate the pH of an aqueous solution of NaOH 1.0x10^{-8} M at 25°C. For full credit make sure that you answer all 6 parts of the problem.

Solution

1) Chemical equation for the complete ionization of NaOH:

\[
\text{NaOH (s)} + xS \text{H}_2\text{O} \rightarrow \text{Na}^+(aq) + \text{OH}^-(aq)
\]

2) If after the addition of NaOH the equilibrium concentration of H_3O^+ ions is y (M), will it be y < 10^{-7} or y > 10^{-7}? (circle one). At equilibrium will it be [OH^-] > 10^{-7} or [OH^-] < 10^{-7} (circle one)

3) Equilibrium (ICE) table for self ionization of H_2O (fill in the blanks)

Chemical equation: 

\[
2\text{H}_2\text{O}(l) \rightleftharpoons \text{H}_3\text{O}^+(aq) + \text{OH}^-(aq)
\]

Initial (M) no self-ionization of H_2O and complete ionization of NaOH:

<table>
<thead>
<tr>
<th></th>
<th>(\text{H}_3\text{O}^+(aq))</th>
<th>(\text{OH}^-(aq))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{H}_2\text{O})</td>
<td>(0)</td>
<td>(1.0 \times 10^{-8})</td>
</tr>
<tr>
<td>(\text{NaOH})</td>
<td>(+y)</td>
<td>(+y)</td>
</tr>
<tr>
<td>(\text{H}_2\text{O})</td>
<td>(-y)</td>
<td>(y + 1.0 \times 10^{-8})</td>
</tr>
</tbody>
</table>

Equilibrium (M) concentrations:

4) Solve the quadratic equation for y:

\[
y^2 + 10^{-8}y - 10^{-14} = 0 \quad \text{or} \quad y = 9.5 \times 10^{-8}
\]

5) Find pH:

\[
\text{pH} = -\log [\text{H}_3\text{O}^+] = -\log y = -\log 9.5 \times 10^{-8} = 7.02
\]

6) Which ionization, the NaOH ionization or the self-ionization of water contributes more OH^- ions to the solution and how many times more?

More OH^- ions: NaOH or H_2O (circle one)

OH^- ions in moles per liter from H_2O = \(9.5 \times 10^{-8}\) M

OH^- ions in moles per liter from NaOH = \(10^{-8}\) M

Therefore: (OH^- from H_2O) = \(Z \text{ times}\) (OH^- from NaOH) with Z = 9.5
Problem

Calculate the pH of an aqueous solution of NaOH $1.0 \times 10^{-8}$ M at 25°C. For full credit make sure that you answer all 6 parts of the problem.

Solution

1) Chemical equation for the complete ionization of NaOH:

$$\text{NaOH}(s) + x5 \text{H}_2\text{O} \rightarrow \text{Na}^+(aq) + \text{OH}^-(aq)$$

2) Before the addition of NaOH, pure water at 25°C has $[\text{H}_3\text{O}^+] = 10^{-7}$ M. After the addition of solid NaOH the solution is $1.0 \times 10^{-8}$ M in NaOH. From $1.0 \times 10^{-8}$ M OH$^-$ added, $y$ (M) of OH$^-$ reacts with H$_3$O$^+$ to reach equilibrium. Will it be $y < 10^{-8}$ M or $y = 10^{-8}$ M? Why?

$K_w = 10^{-14}$ must stay the same.

3) Equilibrium (ICE) table for self ionization of H$_2$O (fill in the blanks)

**Chemical equation:**

$$2\text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{OH}^-(aq)$$

**Initial (M) self-ionization of H$_2$O and complete ionization of NaOH before shifting**

<table>
<thead>
<tr>
<th>Initial (M)</th>
<th>$10^{-7}$</th>
<th>$10^{-7} + 10^{-8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-y$</td>
<td>$-y$</td>
<td>$10^{-7} - y$</td>
</tr>
<tr>
<td>$10^{-7} - y$</td>
<td>$10^{-7} + 10^{-8} - y$</td>
<td>$10^{-7}$</td>
</tr>
</tbody>
</table>

**Change (M)**

**Equilibrium (M) concentrations:**

$$y_1 = 5 \times 10^{-9} \quad , \quad y_2 = 2.05 \times 10^{-7}$$

4) Solve the quadratic equation for $y$:

$$y^2 - 2.1 \times 10^{-7} y + 10^{-15} = 0$$

$$y_1 = 5 \times 10^{-9} \quad , \quad y_2 = 2.05 \times 10^{-7}$$

5) Find pH:

$$[\text{H}_3\text{O}^+]_{eq} = 10^{-7} - y = 10^{-7} - 5 \times 10^{-9} = 9.5 \times 10^{-8} \quad \Rightarrow \quad \text{pH} = 7.02$$

6) Which ionization, the NaOH ionization or the self-ionization of water contributes more OH$^-$ ions to the solution and how many times more?

More OH$^-$ ions: NaOH or H$_2$O (circle one)

OH$^-$ ions in moles per liter from H$_2$O = 9.5 x 10^{-8} M

OH$^-$ ions in moles per liter from NaOH = 10^{-8} M

Therefore: (OH$^-$ from H$_2$O) = Z times (OH$^-$ from NaOH) with Z = 9.5
Method 3

For this very dilute solution, the strong base and the water make comparable contributions to $[\text{OH}^-]$ at equilibrium. Using the principle of charge balance:

$$[\text{OH}^-]_e = [\text{Na}^+]_e + [\text{H}_3\text{O}^+]_e$$

but $[\text{H}_3\text{O}^+]_e[\text{OH}^-]_e = K_w$

so

$$K_w = \frac{[\text{Na}^+]_e + [\text{H}_3\text{O}^+]_e}{[\text{H}_3\text{O}^+]}$$

If you put $K_w = 1.0 \times 10^{-14}$, $[\text{H}_3\text{O}^+] = y$ and $[\text{Na}^+] = 1.0 \times 10^{-8}$ then:

$$y^2 + 1.0 \times 10^{-8}y - 1.0 \times 10^{-14} = 0 \Rightarrow y = 9.5 \times 10^{-8}$$

Which gives again $\text{pH} = 7.02$